Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International Advanced Level

Time 1 hour 30 minutes

Paper reference **WFM03/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Further Pure Mathematics F3

You must have:
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

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Q:1/1/1/1/

P 7 2 4 0 3 A 0 1 3 6

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1. (a) Use the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials to show that

$$\cosh A \cosh B + \sinh A \sinh B \equiv \cosh(A + B)$$

(2)

(b) Hence find the value of x for which

$$\cosh(x + \ln 2) = 5 \sinh x$$

giving your answer in the form $\frac{1}{2} \ln k$, where k is a rational number to be determined.

(5)

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Question 1 continued

Question 1 continued

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Question 1 continued

(Total for Question 1 is 7 marks)

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Determine

$$\int \frac{1}{\sqrt{5 + 4x - x^2}}\,\mathrm{d}x$$

(3)

(ii) Use the substitution $x = 3\sec\theta$ to determine the exact value of

$$\int_{2\sqrt{3}}^{6} \frac{18}{(x^2 - 9)^{\frac{3}{2}}}\,\mathrm{d}x$$

Give your answer in the form $A + B\sqrt{3}$ where A and B are constants to be found.

(6)

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Question 2 continued

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Question 2 continued

(Total for Question 2 is 9 marks)

3.

$$\mathbf{M} = \begin{pmatrix} -2 & 5 & 0 \\ 5 & 1 & -3 \\ 0 & -3 & 6 \end{pmatrix}$$

Given that $\mathbf{i} + \mathbf{j} + \mathbf{k}$ is an eigenvector of $\mathbf{M}$,

(a) determine the corresponding eigenvalue.

(1)

Given that 8 is an eigenvalue of $\mathbf{M}$,

(b) determine a corresponding eigenvector.

(2)

(c) Determine a diagonal matrix $\mathbf{D}$ and an orthogonal matrix $\mathbf{P}$ such that

$$\mathbf{D} = \mathbf{P}^{\mathrm{T}}\mathbf{M}\mathbf{P}$$

(5)

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 8 marks)

4.

$$y = \operatorname{artanh}\left(\frac{\cos x + a}{\cos x - a}\right)$$

where a is a non-zero constant.

Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = k \tan x$$

where k is a constant to be determined.

(4)

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Question 4 continued

(Total for Question 4 is 4 marks)

5. A curve has parametric equations

$$x = 4e^{\frac{1}{2}t} \qquad y = e^{t} - t \qquad 0 \leqslant t \leqslant 4$$

The curve is rotated through 2π radians about the x-axis.

Show that the area of the curved surface generated is

$$\pi(e^{8} + Ae^{4} + B)$$

where A and B are constants to be determined.

(7)

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Question 5 continued

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Question 5 continued

(Total for Question 5 is 7 marks)

6.

$$\mathbf{A} = \begin{pmatrix} x & 1 & 3 \\ 2 & 4 & x \\ -4 & -2 & -1 \end{pmatrix}$$

(a) Show that **A** is non-singular for all real values of x.

(4)

(b) Determine, in terms of x, $\mathbf{A}^{-1}$

(4)

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 8 marks)

7.

$$I_n = \int \frac{x^n}{\sqrt{10 - x^2}}\,\mathrm{d}x \qquad n \in \mathbb{N} \qquad |x| < \sqrt{10}$$

(a) Show that

$$nI_n = 10(n-1)I_{n-2} - x^{n-1}(10 - x^2)^{\frac{1}{2}} \qquad n \geqslant 2$$

(6)

(b) Hence find the exact value of

$$\int_0^1 \frac{x^5}{\sqrt{10 - x^2}}\,\mathrm{d}x$$

giving your answer in the form $\frac{1}{15}\left(p\sqrt{10} + q\right)$ where p and q are integers to be determined.

(4)

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 10 marks)

8. The plane Π has equation

$$3x + 4y - z = 17$$

The line l_1 is perpendicular to Π and passes through the point $P(-4, -5, 3)$

The line l_1 intersects Π at the point Q

(a) Determine the coordinates of Q

(4)

Given that the point $R(-1, 6, 4)$ lies on Π

(b) determine a Cartesian equation of the plane containing PQR

(4)

The line l_2 passes through P and R

The line l_3 is the reflection of l_2 in Π

(c) Determine a vector equation for l_3

(4)

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Question 8 continued

Question 8 continued

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Question 8 continued

(Total for Question 8 is 12 marks)

9. The ellipse E has equation

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

The line l has equation $y = kx - 3$, where k is a constant.

Given that E and l meet at 2 distinct points P and Q

(a) show that the x coordinates of P and Q are solutions of the equation

$$(9k^2 + 4)x^2 - 54kx + 45 = 0$$

(2)

The point M is the midpoint of PQ

(b) Determine, in simplest form in terms of k, the coordinates of M

(3)

(c) Hence show that, as k varies, M lies on the curve with equation

$$x^2 + py^2 = qy$$

where p and q are constants to be determined.

(5)

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Question 9 continued

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Question 9 continued

Question 9 continued

(Total for Question 9 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

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